An Investigation on the Properties of Interferometry

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Using a Helium-Neon laser, four mirrors, and a beamsplitter, we were able to create a Michelson interferometer. We then used it to verify the sinusoidal behavior of the elctromagnetic waves, measure the index of refraction of glass, and examine circular fringes by combining radial wave fronts with planar ones. Our measured value for the index of refraction of the piece of glass we used was $n_g = 1.3427 \pm 0.0024$. This value is consistent with the index of refraction for borosilicate glass which has an index of refraction of 1.36. The fractional difference between our measured value and this value is 1.27%.

I. BACKGROUND/THEORY

The original creation of the Michelson interferometer was actually a failed experiment in trying to prove the "luminiferous aether", a certain medium in which scientist believed light was carried through. Similar to how sound travels through air, scientists assumed that because light obtains wave-like properties then it too should have a medium it travels through. With Michelson's experiment, it was supposed to show that light traveling with the medium, travels faster, and then light traveling against the medium, travels more slowly. Because the projection of the beam after the light reentered the beamsplitter showed a basic interference pattern, this experiment disproved this theoretical aether[2]. More recently, the Michelson interferometer has been used by LIGO in the observation of gravitational waves in confirming "space-time distortion" when dealing with "large scale cosmic events"[3]. Interferometers are mostly used now to "provide sensitivity to changes in optical phase"[1]. Physically, an interferometer dissects a monochromatic source into at least two beams that individually traverse, and are eventually recombined at the beamsplitter. Once recombined, one can observe the beam's interferience characteristics.

After conducting observations on how different aspects of the experiment effect the projected finges, we are able to measure the index of refraction of glass through the number of lapping fringes as a function of the glass's angular rotation. Because the frequency of light remains constant as it passes through a medium, an equation that describes the speed of light through a medium can be described as:

$$v = \lambda f \tag{1}$$

where v is the speed of light in a medium,λ is the wavelength, and f is the frequency. Because we know that the speed of light slows down in a medium, we infer that λ, the wavelength of light, is also able to vary and in this case, shorten increasing the number of light waves present. The relationship between the change in the number of fringes passed and the change in the optical length as defined as $l_{opt} = n \times l_{physical}$ can be mathematically described as

$$\Delta N = \frac{2\Delta l_{opt}}{\lambda} \tag{2}$$

Where l_{opt} is the optical path wavelength number, n is the index of refraction of the material the beam is reflected off of, $l_{physical}$ is the distance of the actual path which we measure, N is the number of fringes, and λ again is the wavelength. The two in equation (2) is due to the fact that the optical path length changes twice [1]. When observing a constructive interference, we can conclude that the two or more waves are completely in sync and therefore the amplitude is two or more times larger. If the waves are completely out of phase, the waves cancel each other out due to destructive interference, meaning no wave. We set up our interferometer as shown in the figure below:

We adjusted the mirrors to translate the beam lateraly, and moved the pitch and yaw in order to align the beam on the screen. Fringes are produced when the wavefronts of the two laser beams, in different phases, combine[1]. The lens was use to magnify the fringe pattern made by the interferometer. Since the waves are in different phase, the wavefronts can combine constructively and destructively. At the point where the two wavefronts intersect inside the beamsplitter, we can get constructive interference, or if the two are out of phate, we can get destructive interference. The fringes appear due to these interference properties. Using derivations of the electric and magnetic field components of the electromagnetic wave, we are able to compare our "sine wave" data collected from the oscilloscope to this to prove the two beams in linear superposition are sinusoidal[1].

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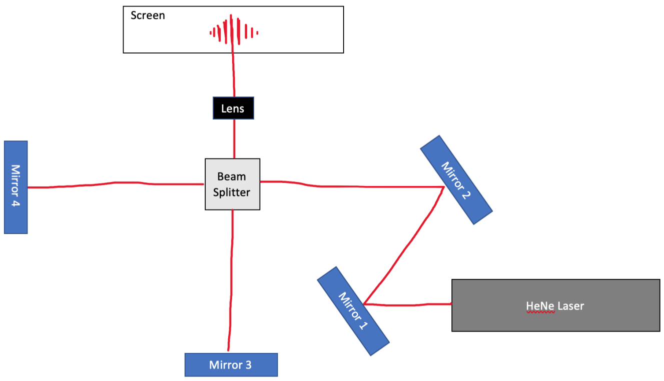

FIG. 1. This is the setup for the lab that we will mainly be using however, we will be putting the lens, L, in a different place. It will either go between the beamsplitter and mirror 3 or mirror 4.[5]

II. UNDERSTANDING THE INTERFEROMETER

In this section of the lab we built an interferometer as described in fig. 1, and used it to verify the sinusoidal pattern of the electromagnetic waves.

A. Measurement Method and Procedure

FIRST, we set up our beamsplitter, four mirrors, screen and HeNe laser as seen in Fig.1. We aligned the mirrors such that when the laser beam bounced off of mirrors 3 and 4, we angled them so that they were hitting the same point of the beamsplitter coming out as they were going in. Once the mirrors were aligned, we then placed a plano-concave lens in between the beamsplitter and the screen in order to magnify the image of the interference pattern on the screen. The following figure is what we saw on the screen.

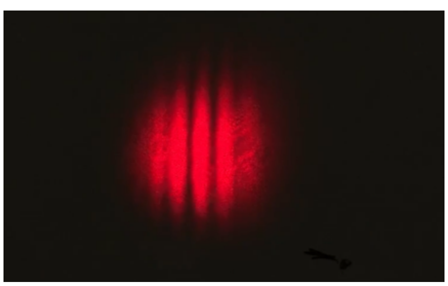

FIG. 2. The vertical lines shown are the interference patterns. The bright spots show areas where there is constructive interference, and the dark spots are destructive interference.

We then move the different arms of the interferometer by moving the mirrors 3 and 4 back and forth. This causes the fringes to move across the screen. When mirror 3 was pushed forward, the fringes as seen in fig.2 would all move together to the left. This is because when one arm is moved, we actually shift the wave fronts of one of the beams, so the points at which the two wavefronts interfere moves to the back or forth and that in turn makes the fringes move to the left or right. When we move mirror 3 away from the beamsplitter, the fringes would move to the right. Conversely, when mirror 4 was pushed forward, the fringes moved to the right, and when we pushed the mirror forward, they moved to the left.
Next, we adjusted the pitch and yaw of mirror 4 until we only saw three fringes appear on the screen. This way when we move the arms, the image 'blinks' as the fringes move left or right. We then removed the lens and focused the laser beam onto a photodiode. The oscilloscope produced sinusoidal images from the photodiode as we would change the length of the arm of the interferometer. We then recorded the images produced and slowed down the video to gain a better understanding of what was going on.

B. Results and Analysis

ONE of the sinusoidal waves that were made from the oscilloscope is shown below:

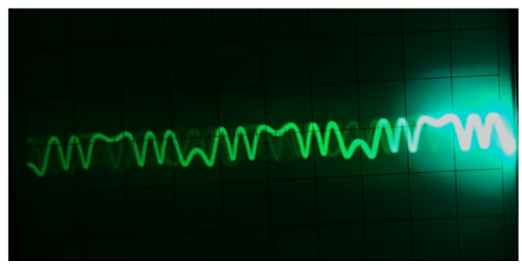

FIG. 3. This is the image that appeared on the oscilloscope when we had the beam focused on it and tapped mirror 4

The change was sinusoidal because electromagnetic waves propagate sinusoidally. And we didn't have just one fringe centered on the photodiode. We actually had multiple fringes, so as they flashed across the photodiode, the different intensities caused different sinusoidal waves to appear. This is evident when you look at the pattern in the wave that repeats itself. This sinusoidal relationship is also seen when combining Ampere's Law and Faraday's Law to create the wave function.

$$E = E_0 cos(kr - \omega t + \phi) \tag{3}$$

$$B = B_0 cos(kr - \omega t + \phi) \tag{4}$$

where E_0 and B_0 are the amplitude vectors, k is the wave number, r is the direction of propagation, ω is the angular frequency, t is the time, and ϕ is the phase shift.

III. INDEX OF REFRACTION MEASUREMENT

In this section we measured the index of refraction of glass through methods of counting passing fringes as a function of the change in angle of the glass with respect to the incoming beam.
Through properties of light and the mechanics of an interferometer, we can use the interferometer to measure the index of refraction of plate-glass. When passing through some medium, the frequency of light is constant, therefor through the relationship of the speed of light through a medium seen in Equation (1). Because light's wavelength through glass is shorter than it is in air, the larger the amount of waves that can fit into the glass' path as the glass' angle gets further from perpendicular to the light source. In this set up, as light passes through the glass plate between one mirror and the beamsplitter, phase is added to the beam because the wavelength is decreased as the beam transmits through the glass.
As we rotate the glass, its thickness, in which the beam travels through, changes. This causes a direct change in the phase, which results in our observation of fringe motion. Through our angle of glass rotation and the number of passing fringes, we are able to measure the index of refraction of our glass [1].

A. Measurement Method and Procedure

USING the set up in Fig. 1., we placed the plate glass (1.35mm $\pm 0.01mm$ thick) and corresponding mounts between the beamsplitter and Mirror 3. We had the plate-glass on a rotating stand so we could change the glass' angle with more ease. By attaching a string to the plate-glass mount, we were able to rotate the glass with precision through means of gently pulling the string. To begin, we set the mirror at 45° to the incident beam. We did this to ensure the beam (λ = 632.8 nm $\pm 0.005nm$ for wavelength) was traveling through the most amount of glass to slow down the speed and wavelength for light. On the wall behind our laser was the projection of the reflected beam from the glass. This reflection is used to calculate the glass' angle when it's rotated.
For calculating θ, we measured l(123.75 cm for trial 1 and 124.65 cm for trial 2 $\pm 0.02cm$), the distance between the glass and the projected image on the wall using a meter stick for the trials. For r, the distance between the original image and the new image, we taped a piece of paper to the wall to mark our original position, then measured the distance between the original beam dot, to the new location after the angle was changed.
To count the fringes, we video taped the fringes in Slow-Mo mode as the glass was being rotated from the string being pulled ever so slightly. We did this to decrease our uncertainty in the fringes.

B. Results and Analysis

IN order to calculate θ we used trigonometry to get:

$$\theta = arctan(\frac{r}{l}) \tag{5}$$

where θ is the angle between the original image and the new image, r is the distance between the two images, and l is the length from the plate-glass and the original image on the wall. Through Equation 5, we were able to calculate our change in angle in relation to the number of passing fringes.
In order to derive an equation for n_g, the index of refraction for glass, we must first see the geometrical relationships as seen in the figure below[1]:

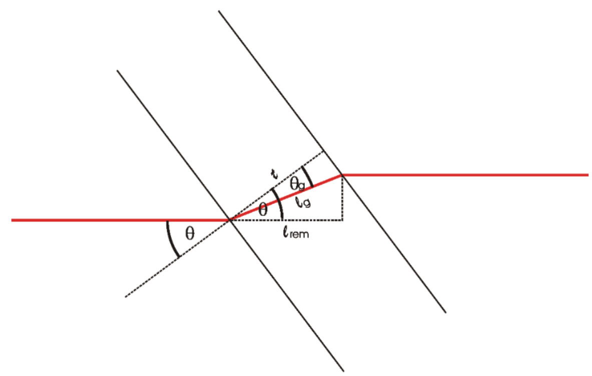

FIG. 4. The light beam has to travel further in this piece of plate glass than it would normally

In this figure 4 above, t is the thickness of the glass, l_{rem} is the length of the path that would've been traveled had the glass not been there, l_g is the length through which the laser travels in the glass. And using Snell's law, the path inside the glass is at an angle of $\theta - \theta_g$.
Through comparison of the number of fringes that pass, ΔN, with the change in optical length of the arm, we get equation(2) where Δl_{opt} is the change in the optical length and λ is the wavelength. The factor of 2 in equation (2) is from the fact that the beam passes through the glass twice (coming out of the beamsplitter and then re-entering). The full optical length is the length of the beam the travels once in between the mirror and the beamsplitter to become:

$$l_{opt} = l_{air} + nl_g \tag{6}$$

where l_{air} is the length light travels in air and l_g is the physical length light travels in the glass. Using trig, we can rewrite l_{opt} to be:

$$l_g = \frac{t}{cos(\theta_g)} \tag{7}$$

where t is the glass thickness and θ_g is the refracted light's angle as it transmits through the front of the glass. Then using Snell's Law we can say:

$$\theta_g = arcsin(\frac{sin(\theta)}{n}) \quad (8)$$

subbing (8) into (7):

$$l_g = \frac{t}{cos(arcsin(\frac{sin(\theta)}{n}} \quad (9)$$

Through substitution we are then able to rewrite l_{opt} and then apply it to equation (2) to get:

$$n = \frac{-sin\theta\sqrt{4t^2 + \Delta N^2\lambda^2 - 4t\Delta N\lambda sin\theta}}{\Delta N\lambda - 2tsin\theta} \quad (10)$$

To calculate for the error in n, we used error propagation techniques learned in Physics 1 Lab. Using our collected data for θ and N, we were able calculate the average index of refraction of our glass to be $n_g = 1.3427 \pm 0.0024$. After conducting further research, we discovered that our glass slide was actually Borosilicate Glass, which is a mix of glass, Silica and boron trioxide, otherwise known as glass and pyrex [6]. Depending on the ratio of each element, the index can range from 1.3-1.6, but since we were dealing with pyrex that is used in chemistry labs and in particular in glass slides, the index is 1.36 which puts us at 1.27%, which is within 1.5% of the actual expected result.

IV. NON-PLANAR PHASE-FRONTS

With the previous set-up, we will replace the glass with a plano-convex lens, "planar side facing the beamsplitter", and remove the two end mirrors (1 and 2 labeled in FIG 1.) [1]. Below is the image produced from making that change:

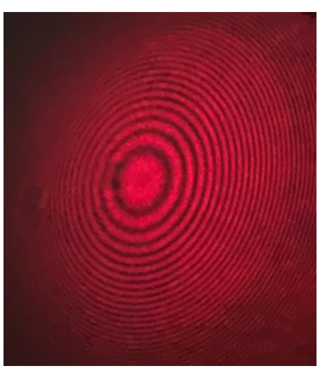

FIG. 5. The circular fringes are caused by the non-planar wavefronts interfering with the planar ones, and the reason we see the circles is because the wavefronts are three dimensional, not just linear.

The plano-convex lens actually provides two beams bouncing back and entering the beamsplitter. One beam bounces off of the convex, curved side, and another beam bounces off of the plane side, and when they both re-enter the beamsplitter, the two beams interact and interfere and create the circular fringes The beautiful image in Fig. 5, was taken with our phone once the setup was all aligned as described.

A. Measurement Method and Procedure

As stated above, we removed the glass and replaced it with a plano-convex lens. We aligned the laser beam by focusing it to hit the same spot on the beamsplitter as it had going in. We also placed a negative focal length lens in between the beamsplitter and screen in order to see a bigger image of the diffraction pattern. We used a $350mm$ focal length pano-conve lens, so we moved the lens to be $3.50cm \pm 0.02cm$ away from the beamsplitter so that the size of both of the beams would be about the same size. Because the beam coming from the planar side is the same size, but the beams that hit the convex side get focused into a focal point and at that point, the two beam sizes are the same. And since we want the beams to interfere when they are the same size, placing the beamsplitter the focal length away form the leans is optimal.

B. Results and Analysis

The path differences for destructive interference points at any point on the projection screen can be characterized and given by [3]:

$$\Delta z = R - \sqrt{R^2 - (x^2 + y^2)} = \frac{x^2 + y^2}{2R} \quad (11)$$

where R is the fringe radius, x and y are the positions of the projection measured on the screen with respect to the center of the rings. We then conclude the relationship to be [3]:

$$\frac{x^2 + y^2}{2R} = \frac{r_m^2}{2R} = m\lambda \quad (12)$$

r_m^2 being $x^2 + y^2$, and λ for the wavelength of the beam, and m is the m^{th} fringe from the center. From this we can compare the center radius (m=1) fringe by $r_1 = \sqrt{2R\lambda}$ so the radius of the fringe changes by $\sqrt{m}r$, where m is the fringe number from the center.

Fringe	Fringe Diameter (cm) ($\pm$0.005cm)	Fringe Thickness (cm)($\pm 0.005cm$)
2	1.38	0.1025
3	1.672	0.238
4	2.001	0.179

Table 1: Analyzed Fringe Data

Our error in Table 1 came from the accuracy of our eye to read precisely down to a certain measurement along with our ability to mark exactly where the fringes occurred. Our collected data was within 95% confidence of the above equations.

C. Above and beyond

After working with the non-polar wavefronts, we wondered what would happen if we tried to change the index of refraction the beam was going through, except not with glass, but with heat instead. So we used a hotplate to warm the air next to the beamsplitter. We used the hotplate and also a thermometer to monitor the change in temperature and carefully watched the fringes to see if anything would happen. At first it was quite difficult to see if any change did actually occur in the fringes. The fringes didn't seem to be moving, but after staring at them for a while, it did seem like they had moved. It would make sense for the fringes to move though, since the index of refraction of hot air is different than that of colder air. And even though it is a smaller change, we thought that we should be able to see something since we do see mirages and stuff like that due to the different index of refraction of hot air. So we used a time-lapse camera to record the fringe pattern and see what change occurred. This yielded results as we expected. We had the time-lapse video record as we changed the temperature from the room temperature of $33°C \pm 0.05°C$ to $130°C \pm 0.05°C$. Every time the temperature raised by $10°C$, we would move our finger into the frame of the camera so that we could see how many fringes moved as we raised the temperature. After reviewing the video, we could clearly see the fringes moving as time increased. As the temperature got hotter, the fringes moved more quickly, and overall we saw about 7 fringes move over the course of the $100°$ temperature shift. So we were able to see and record the minimal change of the index of refraction of hot and cold air which is really cool, especially since the difference index of refraction of air at $0°C$ and $60°C$ is only a minimal change: $n_{0C} = 1.00029238$ and $n_{60C} = 1.00023958$. So when the temperature in air changes by $60°C$, the index of refraction doesn't change until the fifth decimal point.

[1] Dr. Andri M. Gretarsson *Laboratory Optics Manual*

[2] Michelson-Morley / Experiment Introduction,*Khan Acadamy - YouTube* June,2013

[3] Wikipedia, *Michelson Interferometer*

[4] vlab.amrita.edu,*Michelson's Interferometer- Refractive index of glass plate.* June,2013

[5] Michelson Interferometer, PHYWE Systeme GmbH Co.KG - Youtube December,2010

[6] Borosilicate Glass, Wikipedia