Planck's Constant - Measuring h

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In this lab, Planck's constant, h, was measured through the use of the photoelectric effect. Planck's constant is usually applied in quantum mechanics. From the photoelectric effect, electrons are ejected in which kinetic energy is able to be measured. On a cleansed metal surface, photons emit about five different energies/wavelengths. The relationship between the photon's frequency and the electron's kinetic energy is linear, so, with this information, Planck's constant, h, was able to be measured at $(6.2314 \pm 0.07665) \times 10^{-34} \frac{m^2 kg}{s}$, and the work function bounding the electrons to the metal surface, ϕ, was $(2.42695 \pm 0.035815) \times 10^{-19} V$.

I. THEORY

The objective of this experiment was to observe the relationship between an electron's kinetic energy and Einstein's photoelectric equation[3]. The purpose of Planck's constant on radiation can be seen with light, for example. Light "is emitted, transmitted, and absorbed" in small amounts of energy, known as quanta, which can be calculated through the radiation frequency and Planck's constant [2]. From the light energy, electrons are excited which causes them to "be freed" to the surface of the object that the light is hitting. In this lab, a light source passes through five band-pass filters, individually. These filters reduce the range of the radiation frequencies (f) that are allowed to reach the photocell surface. Because these electrons are attached to the metal surface, they are forced to take in the photon in order to overtake to work function (ϕ) which is bounding them to the surface[1]. The energy that is left over after this occurrence is the electron's kinetic energy which can be calculated through[1]:

$$K_{max} = hf - \phi \tag{1}$$

where the kinetic energy is K, the radiation frequency is f, Planck's constant is h, and the work function is ϕ (eV). "The radiation of frequency f can only be emitted in the integral multiples of the basic quantum hf"[1]. The kinetic energy is able to be calculated through an ammeter which measures the current in the cathode plate. Since the maximum kinetic energy is asked for, the maximum voltage, that brings the most "energetic electrons" to rest, is to be measured[1]. The kinetic energy can be calculated through $K_{max} = eV_{max}$[1].

II. PROCEDURE

To begin the lab, 20 minutes was allotted to warm up the mercury lamp. Next, the Picoampere range dial was set to "SHORT", the lid on the mercury light source was removed, and the observing window was opened. Though adjusting the "position of the mercury light source, the photocell unit, and the object glass," the light source was able to be centered in the anode ring, located inside the phototube [3]. After centering, the zeroing dial was set by observation of when the short current reached $0.00 \pm .05\mu A$, then the range selector was set to "Full Scale", and the full scale dial was adjusted until the current read $100.0 \pm 0.05\mu A$. Next the voltage was adjusted until the voltmeter read -1.999$\pm$0.0005V where the uncertainty is half of the last readable value on the Picoampere Amplifier and Control Unit. After this, the range selector was set to $10^{-11} A$.

Starting with the lowest frequency lens, the lens was attached to the objective lens in the phototube and three minutes was given to allow for the current to stabilize. After three minutes, the accelerating voltage was adjusted until the current read $0.00 \pm 0.05\mu A$ and then the voltage value was recorded. Starting with the replacement of the lens, the steps, above, were repeated for each lens in increasing wavelength order.

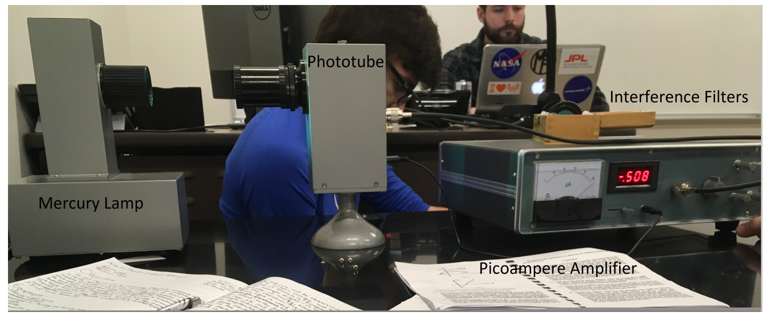

FIG. 1. Above, the mercury lamp is set directly in front of the phototube after the mercury light is centered in the anode ring. On the right is the Picoampere Amplifier and Control Unit with the Intereference Filters resting on top.

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III. RESULTS AND ANALYSIS

To increase the accuracy and precision for the final value of h, two sets of data were taken. The uncertainty in the voltage values, below, are from the precision of the voltage value given on the control unit, how much adjusting the voltage effects the value for the current, and the readability constraints of the human eye during the observation of when the current reads 0.00 μA.

Wavelength (nm)($\pm$0.5nm)	Voltage (V) ($\pm$0.005V)
365.0	1.699
404.7	1.298
435.8	1.134
546.1	0.564
577.0	0.445

Table 1 Voltage Measurements with Varying Wavelengths

The tables above and below feature the recorded accelerating voltages for the corresponding varying wavelengths.

Wavelength (nm)($\pm$0.5nm)	Voltage (V) ($\pm$0.005V)
365.0	1.722
404.7	1.384
435.8	1.165
546.1	0.655
577.0	0.600

Table 2 Voltage Measurements with Varying Wavelengths

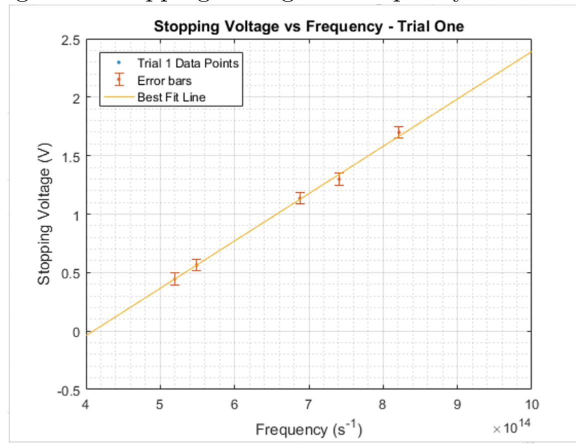
Conversion of the wavelengths into frequency is necessary by using $\lambda = \frac{c}{f}$, where λ is the wavelength, c is the speed of light, and f is the frequency. By plotting the voltage (Y) vs. the frequency (X) and fitting a straight line of best fit to the data, Planck's constant will be able to be found through:

$$\frac{V_{max}}{f} = \frac{h}{e} - \frac{\phi}{e} \tag{2}$$

$$V_{max} = \frac{h}{e} f - \frac{\phi}{e} \tag{3}$$

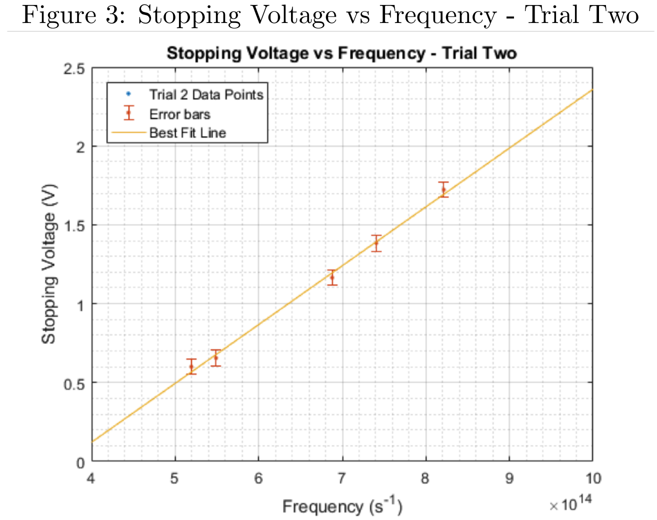
this is because $K_{max} = eV_{max}$[1].

Figure 2: Stopping Voltage vs Frequency - Trial One



The data from Table 1 is shown above. The equation for the graph/line of best fit is $y = 4.0483 \times 10^{-15}x - 1.6589$. After multiplying the $\frac{h}{e}$and $\frac{\phi}{e}$values by the charge of an electron, Planck's constant was calculated to be $(6.4861 \pm 0.0546) \times 10^{-34} \frac{m^2 kg}{s}$ and ϕ was calculated to be $(2.6579 \pm 0.0358) \times 10^{-19} V$. The $\chi^2$for this graph is 1.123 which isn't bad, but isn't the best since there are three degrees of freedom. See Figure 4:χ^2Probability Density Function graph in Error Analysis for further explanation.

Figure 3: Stopping Voltage vs Frequency - Trial Two



The data from Table 2 is shown above. The equation for the graph/line of best fit is $y = 3.7303 \times 10^{-15}x - 1.3707$. After multiplying the $\frac{h}{e}$and $\frac{\phi}{e}$values by the charge of an electron, Planck's constant was calculated to be $(5.9767 \pm 0.0538) \times 10^{-34} \frac{m^2 kg}{s}$ and ϕ was calculated to be $(2.196 \pm 0.03583) \times 10^{-19} V$. The $\chi^2$for this graph is 1.3516 which isn't bad, but isn't the best since there are three degrees of freedom. See Figure 4: χ^2Probability Density Function graph in Error Analysis for further explanation.

By averaging the two h values from Figure 2 and 3:

$$h = (6.2314 \pm 0.0767) \times 10^{-34} \frac{m^2 kg}{s}$$

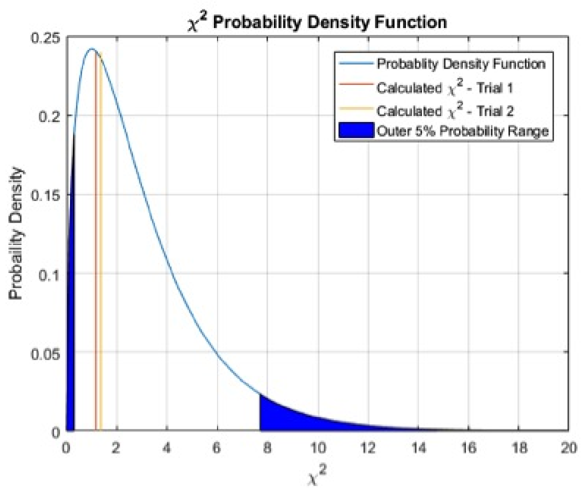
where the uncertainty was averaged through:

$$\delta h = \sqrt{(\delta h_1)^2 + (\delta h_2)^2}.$$

IV. ERROR ANALYSIS

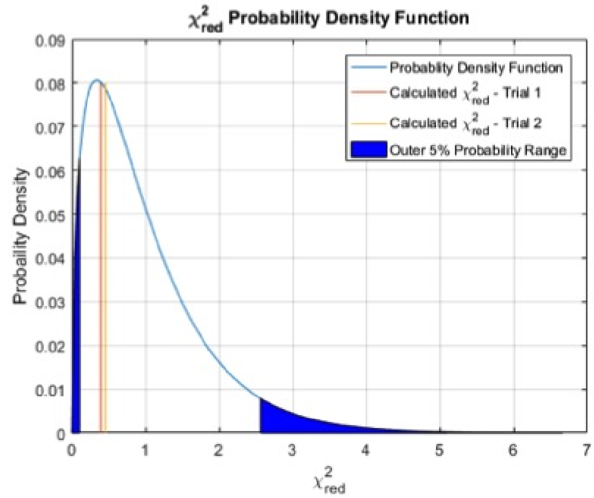
For this lab, the errors were random and systematic. For the random errors, it is difficult to increase the precision of reading the current, voltage, and wavelengths, along with the excessive sensitivity of the picoammeter, therefor, these uncertainties are included into the measurements. As for systematic human errors, it is assumed that "no one is perfect" when it comes to verifying that the needle for the current reads exactly zero and that the phototube stays in the exact same position relative to the mercury during the changing of the lenses, so these uncertainties have also been incorporated into the measurements. In ideal conditions where the mercury lamp and the phototube are stationary, the measured data would increase in accuracy and precision, because the change in orientation between the changes in filters would no longer affect the stopping voltage value.

Figure 4: χ^2Probability Density Function



The probability density function for Table 1 and 2 and Figure 2 and 3 is shown above. With five datum points and 2 parameters, there are three degrees of freedom. The calculated χ^2 value for trial 1 is 1.1623 and for trial 2 is 1.3516 which is good because the values fall inside the 95% confidence area of the probability density function. The reason these values are low is because there were a small number of values to begin with.

Figure 5: χ^2_{red}Probability Density Function



The reduced χ^2 probability density function for Table 1 and 2 and Figure 2 and 3 is shown above. In order to know the results are accurate and precise, the χ^2_{red} should be close to one. The calculated χ^2_{red} for trial 1 is 0.3874 and for trial 2 is 0.4505 which is still okay because these values are within the 95% confidence area. The reason these values aren't very close to one is because there was a small amount of data to go off of.

V. CONCLUSION

The result for this lab is that Planck's constant h is measured to be $(6.2314 \pm 0.0767) \times 10^{-34} \frac{m^2 kg}{s}$. This value is about 5.96% off from the actual accepted value of $(6.62607004 \pm 0.00000081) \times 10{-}34$ J·s[2]. This helps prove the physics in this lab, because, by dividing the maximum voltage required to cause the electron's current to be zero by the filter's frequency, Planck's constant is able to be calculated through Equation 2. As the wavelength of the interference filter increases on the phototube, the voltage, required to bring the electron's current to zero, decreases. This is because fewer radiation frequencies are able to pass through the filter and therefor requires less voltage to slow them down. By clamping or bolting the phototube system to the table to prevent the system from moving during the changing of filters, data for the accelerating voltages would increase in accuracy and precision because the light source's location with respect to the phototube would remain constant.

VI. REFERENCE

(1)Spectrum Techniques
Student Lab Manual
July, 2002

(2)Planck's Constant-Physics
Encyclopedia Britannica

(3)Instruction Manal for Planck's Constant Apparatus Model P67401 Pacific Science Supplies, Inc.