Electron Diffraction

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In this lab, the "interplanar spacing between scattering planes in a polycrystalline graphite lattice" was measured[1]. By observing how electrons can obtain wavelike properties, known as the deBroglie wavelength, one can better understand the characteristics of electron beams. Electrons of equal energy are guided towards a graphite target in which the target has random alignment composing of two main planes of scattering, $d_1 = 0.213 \pm 0.005nm$ and $d_2 = 0.123 \pm 0.005nm$[2]. Because the crystals' planes inside the target have proper alignment, one can observe Bragg diffraction as the beam from the "electron gun" bounces off the crystals. In order to discover the interplanar spacings within the crystals, a fit of the "Bragg scattering angle (θ)" at different voltages is required [1].The interplanar spacings were calculated to be $d_1 = 0.2508 \pm 0.0012nm$ and $d_2 = 0.1464 \pm 0.0004nm$.

I. THEORY

In 1923, Louis de Broglie theorized that all matter could have wavelike properties as well as particle properties. This wave-particle duality was understood with respect to light, but as it turned out, after making various discoveries, it was applicable to any form of matter. He suggested that the wavelength of a particle was related to its momentum and is modeled in the following manner[3]:

$$\lambda = \frac{h}{p} \tag{1}$$

where lambda is the de Broglie wavelength, h is Planck's constant, and p is the momentum. Wavelike properties of electrons are able to be observed through interference patterns as the electrons scatter across successive planes of atoms within the crystalline lattice structure. These patterns and relationships are known as Bragg scattering. For a single crystalline plane, this constructive interference can be expressed in the following fashion[1]:

$$n\lambda = 2dsin(\theta) \tag{2}$$

where d refers to the distance between either diffraction, and θ is the grazing angle measured with respect to the Bragg planes. Therefore, the angle is measured from the surface of the graphite as seen in FIG. 1 [1].

In the special case of a graphite polycrystalline lattice, the atoms are not aligned as linearly as in Figure 1. Instead, they form hexagonal shapes that alter the apparent Bragg planes. This modification is shown in FIG. 2.

The reflected rays or beams of electrons extend outwards in a conical shape which then appears as a ring of light when viewed with a contrasting screen. These light rings are where the visual representation of the constructive interference from the Bragg diffraction can be observed [3].

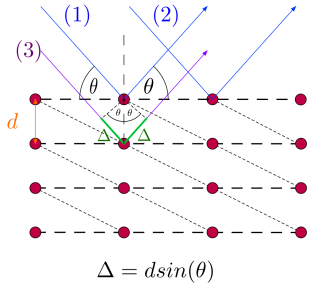

FIG. 1. Bragg diffraction is observed when waves with the same wavelength scatter off of adjacent planes of atoms as shown between rays 1 and 3. Ray 1 and 2 have no path length differences. The path length difference between the two rays 1 and 3 is equal to an integer multiple of the wavelength as shown in Equation (2).[3]

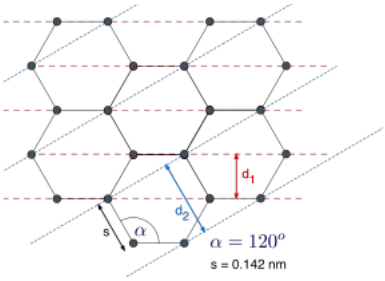

FIG. 2. Shown is the structure of polycrystalline graphite with different Bragg planes indicated by red and blue lines. In this figure, s is the interatomic distance labeled as 0.142nm [3]. In this figure, the lattice spacings are $d_1 = 0.123nm$ and $d_2 = 0.213nm$ [2].

In this experiment an electron diffraction tube is used to observe the scattering. Electrons are accelerated from within the apparatus, towards a graphite target with a potential difference. The kinetic energy of the electrons as they hit the target is modeled by the following relationship [3]:

$$K = \frac{p^2}{2m_e} = eV_a \tag{3}$$

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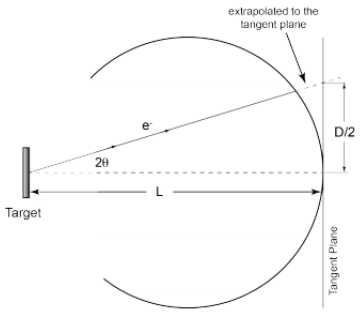

FIG. 3. The geometry of the diffraction tube. In this experiment, $L = 0.140 \pm 0.003m$ to the outside of the glass and the radius $R = 66.0mm$, and the thickness of the glass is 1.5 mm [1].

where K is the kinetic energy and V_a is the potential difference. If the aforementioned deBroglie wavelength equation (Equation 1) is applied to the equation above, then one gets[1]:

$$\lambda = \frac{hc}{\sqrt{2m_e c^2 e V_a}} \tag{4}$$

Then, if all of the known values for h,c, m_e, and e are plugged in, Equation 4 reduces down to [1]:

$$\lambda = \frac{1.227}{\sqrt{V_a}} \tag{5}$$

where the resulting value for λ is in nanometers.

For the second part of the procedure, when the objective is to determine the spacing between planes, some more information must be added. The condition for diffraction at small angles using the equation $\lambda = dsin(\theta)$ becomes[3] :

$$\theta = \frac{D}{2L} \tag{6}$$

where D and L can be understood from Figure 3. Then using equation (5), (6), small angle approximation on equation 2 ($sin(\theta) = \theta$), and the Bragg equation ($2dsin(\theta) = \lambda$), the following equation is derived [3]:

$$d\frac{D}{2L} = \frac{1.227}{\sqrt{V_a}} \tag{7}$$

$$D = \frac{(1.227)(2L)}{d}\frac{1}{\sqrt{V_a}} \tag{8}$$

Then, when plotting D vs $\frac{1}{\sqrt{V_a}}$, the slope value can be used in tandem with Eqn. 8 to find the final values of d, the spacing in between planes of the graphite lattice [3].

II. PROCEDURE

To begin this experiment, the lab was set up as shown in FIG. 4. Next, starting with the voltage knob set at

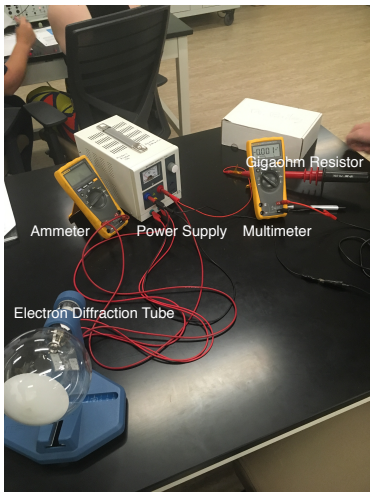

FIG. 4. Above, the Electron Diffraction Tube is connected to the Power Supply and the Ammeter. The Ammeter is wired to the Power Supply and the tube. The Power Supply is connected to the tube, Ammeter, Multimeter, and High Voltage Probe (Resistor). And the Multimeter is wired to the High Voltage Probe (Resistor) and the Power Supply.

$0 \pm 2V$, the voltage was slowly increased until two green rings of different radii appeared. Once the green rings appeared to be prominent, the voltage was recorded and the diameter of both rings was measured using vernier calipers. To verify the measurements were tangent to the tube's screen, the calipers were slightly "hovered over" the rings, which is probably to have an affect on the consistency of the measurements. To increase the accuracy and precision of the measurements, the rings were measured multiple times to help determine the uncertainty in the diameter measurement which was $\pm 0.5cm$. By slowly increasing the voltage, the diameters were recorded for 20 measurements. The uncertainty in the voltage was determined to be $\pm 2V$, because the voltmeter would bounce between two numbers within 2 volts.

III. RESULTS AND ANALYSIS

Accelerating Voltage (V)(± 2 V)	Small Diameter D_i (cm) (±0.05cm)	Large Diameter D_o (cm) (±0.05cm)
2312	3.111	5.260
2603	3.000	4.883
2924	2.750	4.664
3451	2.548	4.332
3626	2.568	4.224
3783	2.432	4.148
3943	2.410	4.032
4092	2.332	3.882
4205	2.344	3.902
4305	2.202	3.978
4439	2.342	3.876
4549	2.268	3.776
4672	2.246	3.764
4781	2.228	3.760
4870	2.200	3.700
4891	2.198	3.363
2385	2.954	5.118
2439	2.952	5.120
2636	2.952	4.774
3313	2.588	4.452

Table 1. From this table, one can observe that as the voltage increases, both of the diameters decrease. With this data, equation (8) is used to plot the diameter of the rings (D) vs $\frac{1}{\sqrt{V_a}}$. With this plot, two lines will be fitted to the data to aid in the determination of the longer and shorter lattice spacings.

With the data from Table 1, a plot of D vs $\frac{1}{\sqrt{V_a}}$ for both D_o (outside) and D_i (inside) can be produced. The reason for the plot being D vs $\frac{1}{\sqrt{V_a}}$ is because when using equation (8), the slope should be $\frac{(1.227)(2L)}{d}$. By fitting the data to equation (8) and using $L = 0.140 \pm 0.003m$[1], two different d values should be able to be found which represent the interplanar spacings of the "honecomb" structure in Figure 5.

From FIG. 5, the line of best fit for the outer ring was $y = \frac{(1.227)2(0.140)}{0.1464}(x) + 0.0032$ where y is the D_o values, x is the $\frac{1}{\sqrt{V_a}}$ values, $d_2 = 0.1464 \pm 0.0004nm$, and the intercept was $C = 0.0032 \pm 0.0001m$. The line of best fit for the inner ring was $y = \frac{(1.227)2(0.140)}{0.2508}(x) + 0.0023$ where y is the D_i values, x is the $\frac{1}{\sqrt{V_a}}$ values, $d_1 = 0.2508 \pm 0.0012nm$, and the intercept was $C = 0.0023 \pm 0.0001m$. The uncertainties in the d and C values were calculated through minimized χ^2. In a perfect lab, the lines of best fit would be expected to intersect with the origin, but because this is not a perfect lab, the intercepts were a tab bit off from intersecting with the origin. This was most likely due to systematic error which will be further explained in the Error Analysis. For the inner ring's fit, the $\chi^2 = 17.8009$ which is pretty good with 20 sets of data and 2 constraints, and the $\chi^2_{red} = 0.9889$ which is very good because this value is very close to 1. For the outer ring's fit, the $\chi^2 = 20.1590$ which is also pretty good with 20 sets of data and 2 constraints, and the $\chi^2_{red} = 1.1199$ which is very good because this value is very close to 1. This data analysis will go in further detail in the Error Analysis.

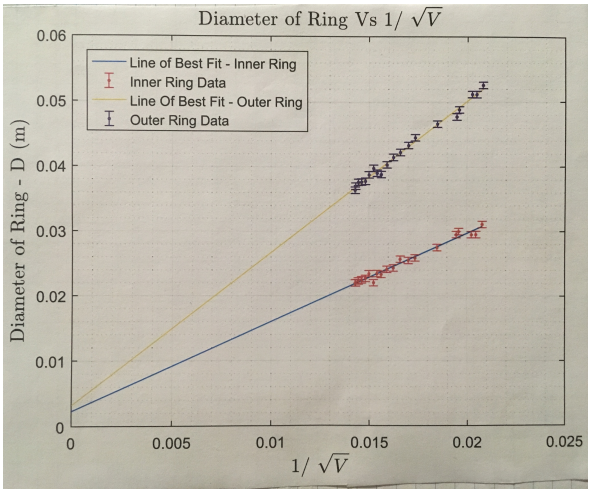

FIG. 5. The graph above shows two lines of best fit with the data from Table 1. The yellow line represents the larger/outer ring and the blue line represents the smaller/inner ring. By plotting the diameter of the rings versus $\frac{1}{\sqrt{V_a}}$, d for the inner and outer ring was able to be determined. The interplanar spacings were found to be $d_1 = 0.2508 \pm 0.0012nm$ and $d_2 = 0.1464 \pm 0.0004nm$ where d_1 represents the outer ring's spacing and d_2 represents the inner ring's spacing.

As a sample calculation using the inner ring's line of best fit:

$$y = \frac{(1.227)2(0.140)}{(0.2508)}(\frac{1}{\sqrt{4092}}) + 0.0023 = 0.0237m \quad (9)$$

The final results for the latttice spacings were that the outer ring was formed from a spacing of $d_2 = 0.1464 \pm 0.0004nm$ and the inner ring was formed from a spacing of $d_1 = 0.2508 \pm 0.0012nm$. From this data, one can conclude that a smaller distance from plane to plane results in a larger ring of photons. Comparing these values to the actual values of $d_2 = 0.123 \pm 0.005nm$ and $d_1 = 0.213 \pm 0.005nm$, the calculated values were accurate out to the tenths place, but the percent difference was about $17 - 19\%$. For the uncertainties in all of the values, a minimized χ^2 was used for each given value in the fit.

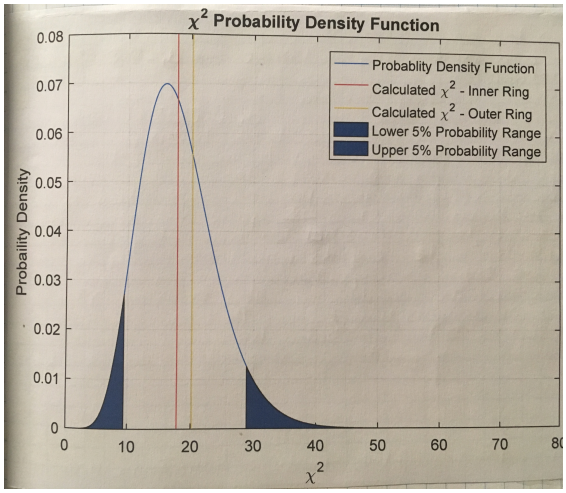

FIG. 6. The probability χ^2 for 18 degrees of freedom for the lines of best fit in FIG. 5 are shown above. The calculated χ^2 for the inner ring's fit was 17.8 and for the outer ring's fit it was 20.2. These values are pretty good because there were 20 sets of data and 2 constraints. Because the χ^2's are well within the 90% confidence range, one can say that the fits to the data are good.

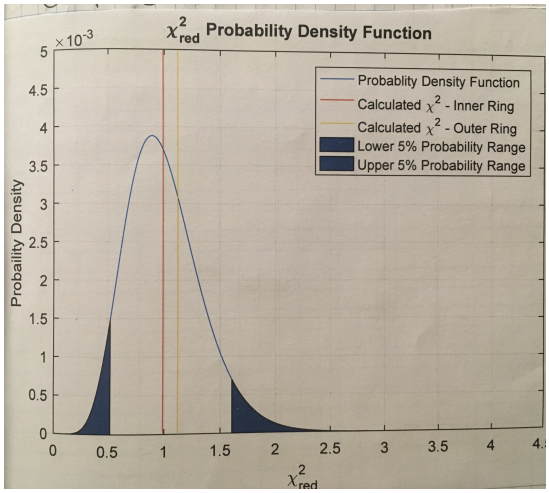

FIG. 7. The reduced χ^2 probability for 18 degrees of freedom for data shown in FIG. 5 is shown above. The calculated χ^2_{red} for the inner ring's fit was 0.9889 and for the outer fit it was 1.1199. Because this is a reduced χ^2, values around 1 characterize a good fit. Since these values are very close to 1, the fit equations are very good and the χ^2_{red} are within the 90% confidence range.

IV. ERROR ANALYSIS

For this lab, the main sources of error were systematic and random error. On the systematic error, it is difficult to increase the precision and accuracy in reading the caliper and multimeter, so this uncertainty is considered in these values as explained in the procedure. As for random error, because the rings were not well defined and it was expected to measure at a tangent plane to the tube's screen, it was more difficult to maintain precise measurements. This aspect from the data pooled the largest amount of uncertainty when it came to measuring the diameter, thus "overshadowing" the uncertainty in the systematic aspect of measuring with the calipers. This is also most likely the cause of the non-zero intercepts in FIG. 5 as well. To better improve this lab, consider including a transparent, thin plane, like laminate, to verify the measurements are conducted on a tangent plane and not around the curved surface of the tube.

V. CONCLUSION

The results from this lab are that the interplanar spacings between scattering planes in a polycrystalline graphite lattice are $d_1 = 0.2508 \pm 0.0012nm$ and $d_2 = 0.1464 \pm 0.004nm$, where d_1 represents the spacings for the inner ring and d_2 represents the spacings for the outer ring. Comparing our values with the accepted and expected values of $d_1 = 0.213 \pm 0.0005nm$ and $d_2 = 0.123 \pm 0.0005nm$, the calculated values are close to the accepted value but differs by $17 - 19\%$ for the inner an outer spacings respectively. The interplanar spacings were able to be calculated by measuring the diameter of the rings at different voltages and plotting and fitting the data to equation (8). This helps illustrate the physics behind electron diffraction, because as the beam of electrons is ejected and accelerated through the graphite target, the electrons act like a wavelength in that they bounce off of different angled crystals which are then projected onto the tube's screen as 2 distinct rings. Through the use of equation (8), it was able to be proven that the electrons have wavelike properties. Both the χ^2 and χ^2_{red} were very good, which means that equation (8) is an accurate representation of the Electron Diffraction Data. As a small addition to this lab, a transparent plane as described in the Error Analysis, may improve the precision and accuracy of the diameter measurements.

VI. REFERENCE

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